P.G. Semester-IV Examination, 2023

MATHEMATICS

Course ID : 42153 **Course Code : MATH-403ME**

Course Title : Modelling and Analysis of Biological Systems

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning unless stated otherwise.

Answer any **five** of the following questions: $8\times5=40$

1. a) Write down the differences between exponential growth equation and logistic growth equation.

   b) Write down the logistic growth equation with maturation delay. Derive the condition(s) for which the delay differential equation exhibits periodic solutions. 2+(2+4)

2. a) Write down a single species harvesting model with proper biological assumptions. Investigate the stability analysis of the equilibrium points of the model and explore the transcritical bifurcation

(if exits) by considering harvesting parameter as the bifurcation parameter.

b) Find the optimal harvesting rate and maximum sustainable yield for the above harvesting model. (2+4)+2

3. a) What is insect outbreak? Write down the Spruce-Budworm model on insect outbreak.

b) Discuss about the occurrence of the saddle node bifurcation in the above model. (2+2)+4

4. In the competition model for two species with populations N_1 and N_2

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - b_{12}\frac{N_2}{K_1}\right)$$
$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - b_{21}\frac{N_1}{K_2}\right)$$

where only one species, N_1, has limited carrying capacity. Non-dimensionalise the system and determine the steady states. Investigate their stability and sketch the phase plane trajectories. Show that irrespective of the size of the parameters the principle of competitive exclusion holds. Briefly describe under what ecological circumstances the species N_2 becomes extinct. 8

5. a) Write down the modified Lotka-Volterra model. Investigate the stability of the equilibrium points.

b) Using Bendixson-Dulac's criteria show that the above system has no close orbit.

c) Using the above results find the conditions for global stability of the interior equilibrium point. (1+4)+2+1

6. Write down the Leslie-Gower model and describe the biological parameters and find biologically feasible equilibrium points and investigate the stability of the model. 8

7. a) Write down a three-species food chain model, where basal prey follows logistic growth, middle predator and top predator follow Holling type-I functional response.

b) Find the existence and stability condition(s) for top predator-free equilibrium point. Give the biological interpretation of the obtained existence and stability condition(s). 2+(4+2)

8. a) Write down the Rosenzweig-MacArthur model with reaction diffusion.

b) Derive the condition(s) for diffusion driven instability (or Turing instability). 2+6

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